EXPERIMENTAL ANALYSIS OF ANTENNA COUPLING FOR HIGH-RESOLUTION DOA ESTIMATION ALGORITHMS

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ABSTRACT

In array signal processing, high-resolution parameter estimation algorithms are known to be sensitive to phase, amplitude and mutual coupling distortions. In this paper, we present experimental results showing that these high-resolution estimation methods can achieve their theoretically expected performances only if the non-ideal array behavior is appropriately modelled and compensated. Using a simple previously proposed distortion compensation technique, we show that it is possible to improve the estimation error considerably, and that actual array response modelling and compensation is indeed an essential element of any high-resolution DOA estimation method.

1. INTRODUCTION

Many computer simulation results confirm the superior performance of high resolution DOA estimation algorithms such as ESPRIT [1,2] and MUSIC [3,4]. In actual arrays, distortions caused by non ideal behaviors (antenna gain, phase and mutual coupling errors) degrade the expected performances of these methods drastically. Usually, these problems are deliberately or unknowingly ignored by many theoreticians. In this work, we present experimental results showing, if these methods have to achieve their theoretical performance, one has to take into consideration the non-ideal array behaviors.

Consider an arbitrary geometry antenna array with M elements being impinged by a single far field signal with the DOA of α. Under non-ideal conditions, the signal at the output of the antenna can be modelled as [5]

$$\boldsymbol{x}(t) = \boldsymbol{C}\boldsymbol{a}(\alpha)\boldsymbol{s}(t) + \boldsymbol{\nu}(t),$$

where $\boldsymbol{x}(t)$ is the measured data vector, $\boldsymbol{a}(\alpha)$ is the ideal array steering vector for a narrow band signal from direction α, $\boldsymbol{s}(t)$ is the signal vector, $\boldsymbol{\nu}(t)$ is a noise vector, and the complex matrix $\boldsymbol{C}$ is a *distortion matrix* that accounts for the combined effects of the antenna gain, phase and coupling errors. If we assmue that the antenna elements have omnidirectional responses, the $\boldsymbol{C}$ matrix is independent of α. Generally, however, it is a function of α. In an actual system, we measure $\boldsymbol{a}_c(\alpha) = \boldsymbol{C}\boldsymbol{a}(\alpha)$, and not $\boldsymbol{a}(\alpha)$. This means that for correct parameter estimation, we need to estimate the distortion matrix $\boldsymbol{C}$.

In the literature, there are a number of works that address this issue. Generally, these methods may be categorized into two classes: blind methods and deterministic methods. The blind approaches, [5–8], are on-line array calibration techniques. They try to estimate both the distortion matrix and the unknown DOAs by making use of some presumed structures. The methods are very appealing, however, they suffer from inconsistency. In fact, it is shown in [9] that these methods give non-unique solutions, when the distortion matrix is allowed to have arbitrary structure. The deterministic methods are off-line array calibration techniques, [9,10]. They compare the estimated array steering vector $\boldsymbol{a}_c(\alpha)$ against the ideal steering vector $\boldsymbol{a}(\alpha)$ to estimate the distortion matrix. The problem of these approaches is that they fail to model the dependence of $\boldsymbol{C}$ on the environment.

The intesion of this work to give experimental results that demonstrate distortion calibration is an essential element in high-resolution DOA estimation methods. We beleive that, a probably better estimate of the distortion matrix may be obtained by combining the two approaches listed in the previous paragraph. That is, to use a deterministic method to get a global approximation of the distortion matrix, and a blind approach to refine the estimate. This is not considered here.

2. MODEL

In this section, we formally develop the data model for a non-ideal array. To be specific, consider a single narrowband signal with DOA of α impinging on a uniform linear array (ULA),

[1] The research of Aweke N. Lemma is supported by the TNO-FEL, The Hague, The Netherlands.

with M elements. Let the N-vector

$$\boldsymbol{x}_m(t)^T = [x_m(t) \;\; x_m(t+1) \;\; \cdots \;\; x_m(t+N-1)]$$

be the t-th sampled signal vector measured at the output of the m-th antenna element. If we assume that there is no mutual coupling between the antennas, $\boldsymbol{x}_m(t)^T$ is given by

$$\boldsymbol{x}_m(t)^T = b_m e^{j2\pi f_o \mu_m} \boldsymbol{s}(t-\tau_m)^T, \qquad (1)$$

where $\boldsymbol{s}(t)^T = [s(t) \;\; s(t+1) \;\; \cdots \;\; s(t+N-1)]$ is the sampled input signal, b_m is the antenna radiation gain, μ_m is the phase distortion in the m-th channel and τ_m is the propagation delay measured from a reference antenna position. For a ULA, we have

$$\tau_m = (m-1)\tau_o \sin\alpha,$$

where τ_o is the time it takes for the signal to propagate between two adjacent antenna elements and α is the DOA measured with reference to the normal of the array axis. Assuming that $s(t)$ is a narrowband signal with a center frequency f_o, (1) can be approximated as

$$\begin{aligned} \boldsymbol{x}_m(t)^T &= b_m e^{-j2\pi f_o \mu_m} e^{-j2\pi(m-1) f_o \tau_o} \boldsymbol{s}(t)^T \\ &=: \gamma_m \theta^{m-1} \boldsymbol{s}(t)^T, \end{aligned}$$

where $\gamma_m = b_m e^{-j2\pi f_o \mu_m}$ and $\theta = e^{-j2\pi f_o \tau_o \sin\alpha}$. Now collecting the M antenna outputs into the $M \times N$ matrix $\boldsymbol{X}(t)$, we get the following model

$$\boldsymbol{X}(t) = \Gamma \boldsymbol{a}(\theta) \boldsymbol{s}(t)^T, \qquad (2)$$

where the complex matrix $\Gamma = \mathrm{diag}\{\gamma_i\}_{i=1}^M$ represents the channel phase distortion and $\boldsymbol{a}(\theta) = [1 \;\; \theta \;\; \cdots \;\; \theta^{M-1}]^T$ is the ideal parameterized array steering vector. In the above model, we have assumed that each antenna element acts independently. In actual case, however, the reflected radiation from one element couples to its neighbors, similar to currents that propagate along the surface of the array. Under this condition, the output of each antenna is the sum of the primary incident signal and the secondary reflected signals from the neighboring elements.

$$\boldsymbol{x}_m(t) = \gamma_m \theta^{m-1} \boldsymbol{s}(t) + \sum_{k=1, k\neq m}^{M} \rho_{m,k} \gamma_k \theta^{k-1} \boldsymbol{s}(t),$$

where the complex factor $\rho_{m,k}$ represents the phase and amplitude of the radiation coupling from the k-th antenna element to the m-th antenna element. Now, collecting the M antenna outputs into an $M \times N$ matrix $\boldsymbol{X}(t)$ as before, we obtain

$$\boldsymbol{X}(t) = (\boldsymbol{I} + \boldsymbol{R}) \Gamma \boldsymbol{a}(\theta) \boldsymbol{s}(t)^T =: \boldsymbol{C} \boldsymbol{a}(\theta) \boldsymbol{s}(t)^T, \qquad (3)$$

where the $M \times M$ matrix $\boldsymbol{R} = \{\rho_{m,k}\}$ is the radiation coupling matrix and $\boldsymbol{I}$ is the identity matrix. From this model, we see that the array steering vector $\boldsymbol{a}(\theta)$ is distorted as the result of the non-ideal array behavior. Consequently, unless the *distortion matrix* $\boldsymbol{C}$ is appropriately compensated, the resulting parameter estimates can be far removed from their true values. In the next section, we shall present a simple technique that can be used to achieve this goal.

3. ESTIMATING THE DISTORTION MATRIX

In this section, we give a simple off-line method of estimating the distortion matrix $\boldsymbol{C}$. The method is similar to those discussed in [9] and [10]. The distinction is that, in our approach, after estimating the array steering vectors we normalize them with respect to their first entries. This gives us a unique solution for the distortion matrix. Note that the solutions described in [9] and [10] are unique up to some complex multiplicative constant which poses problems when working with several independent snapshots. In the following, we assume that the antennas have flat frequency response at the frequency band of interest. Further, we assume that there is only a single source in the channel, though generalization to more sources is possible. We collect the data for Q distinct source positions. Let the data associated with the q-th source position, $1 \leq q \leq Q$, be denoted by $\boldsymbol{X}_q$. Then the distortion matrix $\boldsymbol{C}$ is estimated as follows:

For $q = 1, \cdots, Q$ **Do**

- Collect an $M \times N$ data matrix $\boldsymbol{X}_q$
- Compute the SVD: $\boldsymbol{X}_q = \boldsymbol{U}_q \boldsymbol{\Sigma}_q \boldsymbol{V}_q^H$
- Estimate the signal subspace $\boldsymbol{u}_q$ as the column of $\boldsymbol{U}_q$ that corresponds to the largest singular value.
- Let $\boldsymbol{a}_q$ be the q-th estimated array steering vector and let $\boldsymbol{a}_q(m)$ be its m-th entry, then set

$$\boldsymbol{a}_q(m) = \boldsymbol{u}_q(m)/\boldsymbol{u}_q(1), \;\; m = 1, \cdots, M.$$

- Construct an augmented array response matrix $\boldsymbol{A}_c$ as

$$\boldsymbol{A}_c = [\boldsymbol{a}_1 \;\; \cdots \;\; \boldsymbol{a}_Q]$$

End Do

Finally, compute the distortion matrix as

$$\boldsymbol{C} = \boldsymbol{A}_c \boldsymbol{A}^H (\boldsymbol{A}\boldsymbol{A}^H)^{-1}, \qquad (4)$$

where $\boldsymbol{A} = [\boldsymbol{a}(\theta_1) \;\; \cdots \;\; \boldsymbol{a}(\theta_Q)]$ is a matrix containing the Q true array steering vectors corresponding to the Q source positions. Note that, for this method to work Q must be greater or equal to the number of antenna elements M.

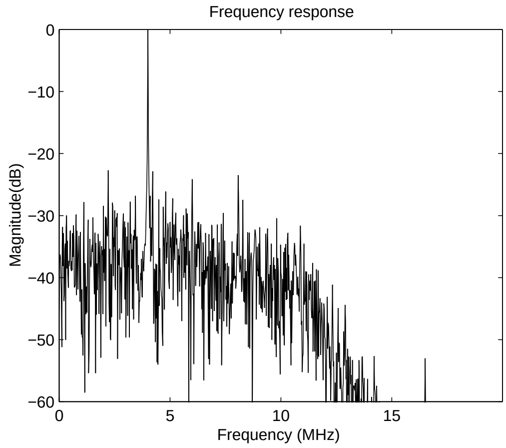

Figure 1: The frequency spectrum of the received signal

4. EXPERIMENTAL RESULTS

The experiment we report here was conducted at an outdoor location by TNO-FEL, Netherlands. The channel behavior fits into a typical rural area scenario. A narrowband signal source with a center frequency of 108.9 MHz was placed at a distance of approximately 260 m (290 yards) from a ULA with M=4 omnidirectional anetenna elements and a baseline length of 109 cm. The antenna array was mounted on a rotating table, such that with a fixed source position, it was possible to generate different angles of arrivals by varying the orientation of the antenna array. The received signal was down converted to an IF frequency with a local oscillator operating at 104.9 MHz. After lowpass filtering with a cut-off frequency of 10 MHz, the signal was then sampled at 40 MHz (2 times the Nyquist rate). As can be seen from the frequency spectrum of the received signal in Fig. 1, the measurement conditions indeed fit into a single source scenario, with a SNR of approximately 25 dB.

The measurements were conducted for DOAs varying from -75 to 75 degrees with steps of 15 degrees. The distortion matrix was estimated with the procedure described in section 3 making use of the measurements corresponding to the DOA $= [-75 \quad -30 \quad 0 \quad 45 \quad 75]$ degrees. Then, the ESPRIT $[1, 2]$ and MUSIC $[3, 4]$ algorithms were applied to estimate the DOAs for each measurement. The performance improvements of the distortion compensated approach over the direct one are summarized in figures 2 and 3 for the MUSIC and the ESPRIT algorithms, respectively. Note that in the MUSIC approach, the DOAs are obtained by searching spectral peaks in the MUSIC spectrums, whereas in the ESPRIT algorithm, the DOAs are computed from phase estimates obtained via shift invariance considerations. As a final result, in table 1, the true DOAs and the estimated DOAs (before and after distortion compensation) are summarized, and the corresponding biases in degrees as functions of the DOA are shown in Fig. 4.

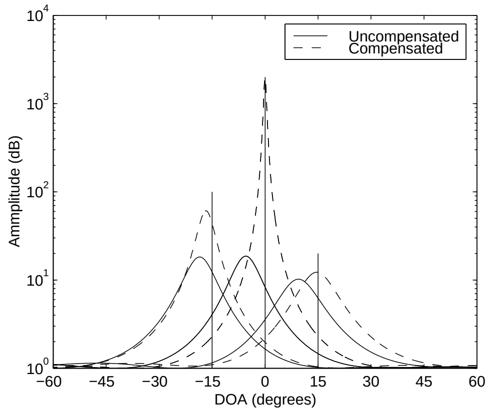

Figure 2: The MUSIC spectra, for three DOAs. The true DOAs are indicated by solid vertical lines

Table 1: Effect of distortion compensation on DOA estimates (single source)

Expected	Uncompensated		Compensated	
	ESPRIT	MUSIC	ESPRIT	MUSIC
75	87.5	87.5	75.4	74.8
60	71.9	65.1	59.8	59.0
45	47.9	45.4	45.5	43.6
30	27.8	28.0	31.3	30.0
15	12.0	9.4	13.6	14.6
0	-5.6	-5.5	0.0	0.0
-15	-20.7	-18.5	-16.5	-16.7
-30	-35.5	-30.6	-30.3	-30.2
-45	-60.4	-47.1	-44.8	-45.0
-60	-92.5	-67.4	-59.4	-60.3
-75	-92.5	-92.5	-75.9	-76.3

In the second measurement, the experiment was repeated for a two source case, where two narrow band signals with IF center frequencies 4 and 5 MHz and DOAs 75 and 60 degrees, respectively, were considered. The MUSIC and ESPRIT results are summarized in table 2

5. CONCLUSION

In this experimental work, we have shown that, if high-resolution DOA estimation algorithms are required to achieve their theoretically expected performances, it is mandatory to appropriately compensate the distortions caused by non-ideal array behaviors. Here, a simple off-line compensation method was employed. For better results, it may be useful to fine-

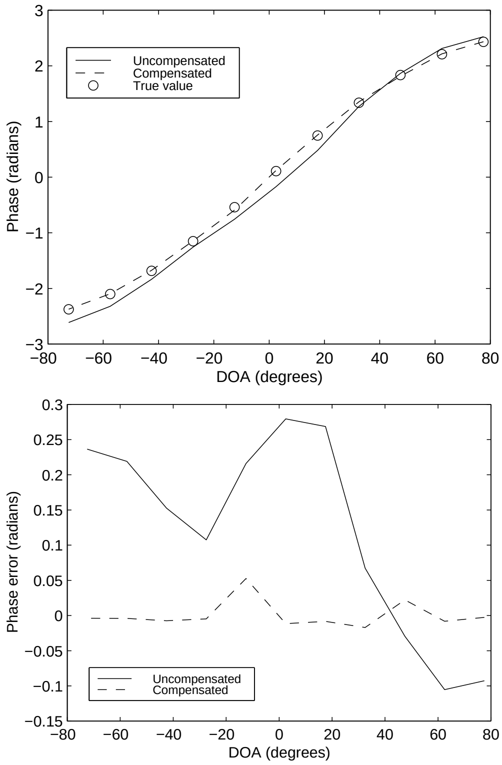

Figure 3: a) ESPRIT phase behaviors as functions of DOA, and b) Errors in the phase estimates before compensation (solid line) and after compensation (dashed line)

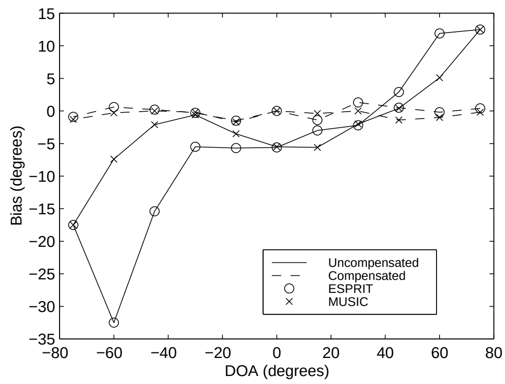

Figure 4: The biases in degrees between the measured and true DOAs before and after the compensations

tune the distortion matrix using on-line blind compensation algorithms such as those described in [5–8]. The main message of this experimental report is that any practical solution to parameter estimation problem should address the underlying actual system behaviors, such as phase, amplitude and coupling distortions.

Table 2: Effect of distortion compensation on DOA estimates (two sources)

Expected	Uncompensated		Compensated	
	ESPRIT	MUSIC	ESPRIT	MUSIC
75	77.9	78.3	74.6	72.9
60	68.6	70.8	62.6	63.1

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