INTERPOLATION-BASED MULTI-MODE PRECODING FOR MIMO-OFDM SYSTEMS WITH LIMITED FEEDBACK

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ABSTRACT
Spatial multiplexing with multi-mode precoding provides a means to achieve both high capacity and high reliability in Multiple-Input Multiple-Output Orthogonal Frequency Division Multiplexing (MIMO-OFDM) systems. Multi-mode precoding uses linear transmit precoding that adapts the number of spatial multiplexing data streams or modes, according to the channel conditions. To do so, it typically requires complete knowledge of the transmit precoding matrices for each subcarrier at the transmitter. In this paper, we propose to reduce the feedback requirements by sending back the quantized right singular vectors of the MIMO channels at a fraction of the subcarriers and interpolating between them. Mode selection is performed at the receiver and the decisions are sent back to the transmitter. An interpolation algorithm is presented that is based on interpolation in the Stiefel manifold. Bit-Error Rate (BER) performance simulations demonstrate the performance improvements provided by the proposed algorithms as a function of the feedback rate.

1. INTRODUCTION
To achieve high spectral efficiency and high link reliability over highly space- and frequency-selective wireless channels at a reasonable hardware cost, Multiple-Input Multiple-Output (MIMO) Orthogonal Frequency-Division Multiplexing (OFDM) facilitates MIMO processing at a per subcarrier basis, by converting a frequency-selective MIMO channel into a number of parallel flat fading MIMO channels. In particular, spatial multiplexing with linear precoding, realizes significant performance improvements compared to traditional spatial multiplexing, by improving its robustness against rank deficiencies of the MIMO channel [1, 2]. Unfortunately, to realize its full gains, spatial multiplexing with linear precoding requires full Channel State Information (CSI) at the receiver, and an assumption, which does not hold for any practical system. In this perspective, recent research works have considered more realistic partial transmit CSI scenarios in the form of first/second order channel statistics [3, 4, 5], however, at the expense of reduced performance.

Basically, two fundamentally different approaches can be pursued to solve the transmit CSI acquisition problem: the open-loop and the closed-loop approach. On the one hand, the open-loop approach, which is best suited for Time-Division Duplexing (TDD) systems, exploits the CSI estimated in one communication direction for the calculation of the precoder employed in the other communication direction. However, to restore the channel reciprocity between up- and downlink, which is destroyed by amplitude and phase mismatches between the multi-antenna Radio Frequency (RF) transceiver branches, this approach calls for either expensive front-end calibration circuitry [18], or bandwidth consuming over-the-air calibration procedures [19]. On the other hand, the closed-loop approach, which is applicable to both TDD and Frequency-Division Duplex (FDD) systems, estimates the CSI at the receiver side and conveys it to the transmitter side through a feedback channel [6, 7]. However, one of the major challenges for this approach is to come up with a practical solution that incurs minimal feedback overhead, which is exactly the scope of this paper.

The natural approach is to quantize the MIMO channel directly, and convey the quantized channel coefficients back to the transmitter [9]. Unfortunately, direct quantization of the channel matrix is not efficient for large numbers of transmit and receive antennas, since high feedback rates are required to minimize the quantization error. On the other hand, quantization of the precoder (as opposed to the channel) allows for a better compression by exploiting the structure of the precoding matrix [6]. In this paper, we follow the latter approach, and, more specifically, the codebook framework of [6], in which the precoding matrix is selected from a pre-determined codebook of precoding matrices. Previously proposed codebook designs [6, 15], however, do not lend themselves to efficient multi-mode optimization for MIMO-OFDM systems. Nevertheless, multi-mode optimization, by adapting the number of spatial multiplexing streams according to the channel conditions, enables significant performance improvements over its un-optimized counterparts [14].

To enable an efficient multi-mode optimization for MIMO-OFDM systems, we propose to construct a precoder codebook, corresponding to the quantization of the complete right singular matrices of the MIMO channels on the different subcarriers. Furthermore, we reduce the feedback requirement, by only conveying the optimal indexes of the codewords on a fraction of the subcarriers, so-called pilot subcarriers. Then, to recover the precoders on the remaining subcarriers, we present a novel interpolation al-
2. SYSTEM MODEL

We consider a spatial-multiplexing MIMO-OFDM wireless communication system that consists of an $M_T$-antenna transmitter, an $M_R$-antenna receiver and $N$ subcarriers. On the $k^{th}$ subcarrier, the transmitter optimally maps the $M_s[k]$-dimensional spatial data vector $s[k] = [s_1[k] \cdots s_{M_s[k]}[k]]^T$, where $M_s[k] \leq \text{Min}(M_T, M_R)$, onto the $M_T$ transmit antennas using a linear precoder $F[k]$. This linear precoder $F[k]$ is designed based on quantized CSI acquired through a low-rate feedback channel. If the cyclic prefix is larger than the channel length, the linear convolution with the frequency-selective MIMO channel is observed as cyclic. Thus, on the $k^{th}$ subcarrier, it becomes equivalent to multiplication with the discrete Fourier transform of the MIMO channel, given by the $M_T \times M_T$ channel matrix $H[k]$, whose entries represent the channel gains experienced by subcarrier $k$. Consequently, the $M_R$-dimensional received signal vector, on the $k^{th}$ subcarrier, $y[k]$ is given by:

$$y[k] = H[k]F[k]s[k] + n[k]$$  \hspace{1cm} (1)\n
where $n[k]$ is the $M_R$-dimensional zero-mean spatially-white complex Gaussian receiver noise vector of covariance matrix $N_0 I_{M_R}$ and $E\{s[k] s[k]^H\} = \frac{E_s}{m_{s[k]}} I_{M_s[k]}$. Clearly, OFDM modulation decouples the convolutional MIMO channel into a set of $N$ orthogonal flat-fading channels, on the $N$ subcarriers. This property is exploited to carry out data detection on each subcarrier independently. Accordingly, on subcarrier $k$, $s[k]$ is detected using the MMSE receiver $G[k] = \left( \frac{M_s[k]}{E_s} I_{M_s[k]} + F[k]^H H[k] F[k] \right)^{-1} F[k]^H H[k]^H$.  

3. OVERVIEW OF THE PROPOSED MULTI-MODE PRECODING ARCHITECTURE

In this contribution, every linear precoding matrix $F[k]$, on the $k^{th}$ OFDM subcarrier, is assumed to have orthonormal columns; $F[k]^H F[k] = I_{M_s[k]}$. This follows from the form of the optimal precoder solutions with full CSI derived in [1, 2]. Consequently, power loading across the spatial multiplexing streams is skipped because they lead to only a marginal performance improvement while significantly increasing the feedback requirement [6]. Furthermore, the transmit power is evenly distributed among the $N$ subcarriers.

Let $H[k] = U[k]\Sigma[k]V[k]^H$ be the Singular Value Decomposition (SVD) of the MIMO channel on the $k^{th}$ subcarrier, where $U[k]$ and $V[k]$ are respectively $M_T \times M_R$ and $M_T \times M_T$-dimensional matrices containing the left and right singular vectors associated to the singular values or modes of $H[k]$. These modes are stacked in decreasing order in the $M_R \times M_T$ diagonal matrix $\Sigma$. When perfect CSI is available at the transmitter and the receiver is linear, it is well-known [1, 2, 6] that the optimal precoder $F[k]$ consists of the $M_s[k]$ first columns of $V[k]$, i.e. $F[k] = [V[k] \cdots V[k]]_{1 : M_s[k]}$, where $M_s[k]$ is the number of spatial multiplexing data streams to be transmitted on the $k^{th}$ subcarrier. In this contribution, however, the transmitter does not perfectly know the CSI on each subcarrier $k$ but there exists a low-rate feedback channel from the receiver to convey information about each optimal $F[k]$. This feedback channel is herein exploited to deploy the quantized precoding approach of [6], where the precoding matrix on each subcarrier $k$ is selected from a pre-determined codebook of precoding matrices $\mathcal{F}[k] = \{F_1[k], \cdots, F_{\text{card}(\mathcal{F}[k])}[k]\}$, known a priori to both the transmitter and the receiver. In this approach, the receiver selects, based on the perfect and complete CSI, $H[k]$, the optimal precoding matrix from $\mathcal{F}[k]$ and conveys only the particular index of the selected precoding matrix to the transmitter through the feedback channel. For simplicity, we consider the problem of independently quantizing the precoding matrices on the different subcarriers. This independent quantization approach, combined with the observation that the MIMO channels on the different subcarriers exhibit the same statistics, leads to a single codebook $\mathcal{F} = \{F_1, \cdots, F_{\text{card}(\mathcal{F})}\}$ for all subcarriers. Furthermore, we subsequently propose to reduce the feedback through conveying the information only about a limited set of $U$ pilot subcarriers and recovering the precoders on the remaining subcarriers through interpolation. This interpolation-based approach, combined with the constraint of supporting multi-mode precoding, does not allow the use of the Grassmannian codebook designs proposed in [6] because the unitary ambiguity that they exhibit, compromises the ordering of the channel singular modes, which is crucial for the deployment of multi-mode quantized precoding at the transmitter. The multi-mode codebook design proposed in [15] cannot be used either, in conjunction with precoder interpolation, because the interpolated precoders may exhibit larger optimal number of spatial streams than the used pilot precoders. Consequently, we alternatively propose to construct a precoder codebook corresponding to the quantization of the complete $M_T \times M_T$-dimensional unitary right singular matrix $V[k]$. Furthermore, we propose to feedback the indexes of the codewords corresponding to the quantized precoders on $U$ pilot subcarriers $\{F[k]\}_{1 \leq j \leq U}$, interpolate these pilot precoders to recover the precoders on the remaining subcarriers and then enforce the optimal number of modes on each subcarrier, as determined and feedback from the receiver. The quantization of the square unitary matrices $\{V[k]\}_k$ and the corresponding construction of the square precoder codebook $\mathcal{F}$ are discussed in Section 4. The interpolation as well as mode selection are subsequently discussed in Section 5 and Section 6 respectively. The simulation results in Section 7 assess the performance of the proposed interpolation-based multi-mode quantized precoding. Finally, Section 8 draws the final conclusions.

4. DESIGN OF THE CODEBOOK $\mathcal{F}$ OF PER-SUBCARRIER QUANTIZED PRECODERS

Our strategy is to parametrize the unitary matrices $\{V[k]\}_k$ and then quantize the parameter space. We consider the parametrization obtained by decomposing a unitary matrix into Givens rotations as detailed in [7] because it requires a minimum number of parameters and it leads to a one-to-one mapping. This parametrization represents an $M_T \times M_T$-dimensional complex unitary matrix
\[ \mathbf{V} = \prod_{k=1}^{M_T} \mathbf{D}_k(\phi_{k,k}, \ldots, \phi_{k,M_T}) \prod_{i=1}^{M_T-k} \mathbf{Q}_{M_T-l, M_T-l+1}(\theta_{k,i}) \mathbf{I}_{M_T} \]

where the \( M_T \)-dimensional diagonal matrix

\[ \mathbf{D}_k(\phi_{k,k}, \ldots, \phi_{k,M_T}) = \text{diag}(1_k, e^{j\phi_{k,k}}, \ldots, e^{j\phi_{k,M_T}}) \]

where \( 1_k \) is \((k-1)\)'s, \( \mathbf{Q}_{p-1,p}(\theta) \) is the Givens matrix which operates in the \((p-1,p)\) coordinate plane of the form:

\[ \mathbf{Q}_{p-1,p}(\theta) = \begin{bmatrix} 1_{p-2} & \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \\ \end{bmatrix} \mathbf{I}_{M_T-p} \]

Based on this parametrization, we propose to quantize the set of unitary matrices \( \{\mathbf{V}[k]\}_k \) through jointly determining the reconstruction points for all quantized parameters \( \{\phi_{k,j}, \theta_{k,i}\}_{k,j,i} \) using the LBG algorithm for vector quantizer design [16] with a squared error distortion. Then, the codebook of unitary matrices, \( \mathcal{F} \), is designed by reconstructing unitary matrices from the quantized parameters \( \{\phi_{k,j}, \theta_{k,i}\}_{k,j,i} \).

5. PRECODER INTERPOLATION FOR MIMO-OFDM

As previously announced, the above-introduced quantized codebook, \( \mathcal{F} \), is now used to efficiently feedback the \( M_T \times M_T \)-dimensional precoders on a limited set of \( U \) pilot subcarriers. Based on these pilot precoders, we subsequently seek to recover the precoders on the remaining subcarriers through interpolation. This is reasonable because the frequency correlation exhibited by the MIMO channels across subcarriers was shown [8] to hold for the linear precoders on these subcarriers. More specifically, we propose a so-called geodesic interpolation strategy that exploits this frequency correlation or smoothness to interpolate the available unitary quantized pilot precoders \( \{\mathbf{F}[k]\}_{1 \leq k \leq U} \) on the Stiefel manifold [10], i.e. under a unitary constraint. These pilot precoders are assumed to be equidistantly spaced across the \( N \) subcarriers.

The geodesic approach considers each 2 unitary precoders on successive pilot subcarriers, for instance \( \mathbf{F}[k_i] \) and \( \mathbf{F}[k_{i+1}] \) with \( 1 \leq i \leq U-1 \), and interpolates to recover the unitary precoders on all subcarriers in between. More specifically, it considers these 2 pilot precoders as 2 frames on the Stiefel manifold and tries to identify the smoothest trajectory on the Stiefel manifold between these 2 frames. The rotations constructing this trajectory, referred to as a geodesic, are the desired interpolated unitary precoders on the subcarriers between the 2 successive pilots. This so-called Stiefel interpolation is widely known in the computer vision literature [11, 12], where it is the optimal way to perform grand tours of 3-D objects. Before detailing the proposed interpolation solution, we highlight that every right singular matrix \( \mathbf{V}[k] \), and by extension its quantized version \( \mathbf{F}[k] \), is ambiguous up to a diagonal unitary matrix \( \Theta[k] \), which determines the orientation of the right singular vectors. In fact, each orientation matrix \( \Theta[k] \) represents \( M_T \) additional degrees of freedom, which we propose to exploit in order to optimize the proposed interpolation solution.

5.1. Optimization of the orientation matrices

We propose to optimize the orientation matrix \( \Theta[k_i] \) related to the \( 2 \) successive pilot precoders \( \mathbf{F}[k_i] \) and \( \mathbf{F}[k_{i+1}] \), with \( 1 \leq i \leq U-1 \), such that the 2 frames \( \mathbf{F}[k_i] \) and \( \mathbf{F}[k_{i+1}] \Theta[k_i] \) are as close as possible in Frobenius norm:

\[ \Theta[k_i] = \min_\Theta \| \mathbf{F}[k_i] - \mathbf{F}[k_{i+1}] \Theta[k_i] \|_F \]

where \( \mathbf{D} \) is diagonal and unitary. This additional optimization aims at identifying the optimal orientation of the quantized singular vectors in \( \mathbf{F}[k_{i+1}] \) that minimizes their Euclidean distance to \( \mathbf{F}[k_i] \), such that the subsequent geodesic interpolation can then be successfully used to identify the smoothest path between these 2 pilot precoders. Let \( \mathbf{A}[k_i] = \mathbf{F}[k_{i+1}]^{-1} \mathbf{F}[k_i] \), it can be easily shown that the optimal orientation matrix \( \Theta[k_i] \) is given by:

\[ \Theta[k_i] = \text{diag} \left( \frac{A_{1,1}[k_i]}{A_{1,1}[k_i]}, \ldots, \frac{A_{1,1}[k_i]}{A_{1,1}[k_i]}, \ldots, \frac{A_{M_T,M_T}[k_i]}{A_{M_T,M_T}[k_i]} \right) \]

Our proposed solution, to the problem of optimizing the orientation matrices, exhibits the attractive feature of solely depending on the quantized pilot precoders, as such it can be carried out at the transmitter and does not require additional feedback.

5.2. Geodesic interpolation

Capitalizing on the previous optimization, we now apply the geodesic interpolation to \( \mathbf{F}[k_i] \) and \( \mathbf{F}[k_{i+1}] \Theta[k_i] \), instead of the original version that simply interpolated between \( \mathbf{F}[k_i] \) and \( \mathbf{F}[k_{i+1}] \). To do so, we first make the following transformation on the pilot frames, to position the start frame on the identity element of the Stiefel manifold, where the geodesic is known:

\[ \begin{cases} \mathbf{F}[k_i] & \rightarrow \mathbf{I}_p \\ \mathbf{F}[k_{i+1}] \Theta[k_i] & \rightarrow \mathbf{M} = \mathbf{F}^{-1}[k_i] \mathbf{F}[k_{i+1}] \Theta[k_i] \end{cases} \]

It was shown that the geodesic (tangent at the identity element) is defined as:

\[ \Phi_t(t) = \exp(S) t \in [0, 1] \]

where \( S \) is skew-hermitian (i.e. \( S^H = -S \)) and \( \mathbf{M} = \exp(S) = \Phi_1(1) \). This form is know as the exponential map of the unitary matrix \( \mathbf{M} \). In fact, every unitary matrix can be written on that form where the matrix exponent is skew-hermitian [11, 12]. In order to determine \( S \) starting from \( \mathbf{M} \), we use the eigenvalue decomposition:

\[ \mathbf{M} = \mathbf{A} \Sigma \mathbf{A}^{-1} \]

Since \( \Sigma \) is a diagonal matrix, we can easily define its exponential map \( \Sigma = \exp(\Sigma) \). Consequently, \( \mathbf{M} \) of (9) can be re-written as:

\[ \mathbf{M} = \mathbf{A} \exp(\Sigma) \mathbf{A}^{-1} \]

Finally, we can determine the skew-hermitian matrix of the exponential map of \( \mathbf{M} \) in (8) as \( \mathbf{S} = \mathbf{A} \Sigma \mathbf{A}^{-1} \). After having determined the exponential map of \( \mathbf{M} \), we can reverse the initial transformation of the pilot frames in (7) and consequently identify the geodesic or set of rotations between \( \mathbf{F}[k_i] \) and \( \mathbf{F}[k_{i+1}] \Theta[k_i] \) as:

\[ \Phi_t(t) = \mathbf{F}[k_i] \Phi_t(t) = \mathbf{F}[k_i] \exp(t\mathbf{S}) \ t \in [0, 1] \]

where \( \mathbf{S} \) is given by (10) and the step in the definition of \( t \) is determined by the number of subcarriers between the 2 successive pilot subcarriers.
6. MODE SELECTION FOR MIMO-OFDM WITH LIMITED FEEDBACK

So far, we have interpolated the \( M_T \times M_T \) unitary pilot precoding matrices \( \{F[k_i]\}_{1 \leq i \leq U}, \) under a unitary constraint, to recover the \( M_T \times M_T \) unitary precoding matrices on all subcarriers \( \{F[k]\}_{1 \leq k \leq N}. \) Our strategy now is to select the \( p_{opt}[k]-\)first columns of each interpolated \( F[k] \) matrix to instantiate the optimal spatial multiplexing mode, where \( p_{opt}[k] \) is the optimal number of spatial streams to be used on the \( k^{th} \) subcarrier. The optimality pertains to the minimization of an upper-bound on the symbol-vector error rate [14, 15], which was shown to be achieved through the maximization of the Signal-to-Noise Ratio (SNR) on the weakest spatial stream [13]. More specifically, the mode-selection optimization criterion, for the \( k^{th} \) subcarrier, reads:

\[
\begin{align*}
    p_{opt}[k] &= \max_{M_{opt}[k]} \left\{ \lambda_{\min}(H[k]F_{1:M_{opt}[k]}(k)M_{opt}[k]) \right\} \\
    p_{opt}[k] \cdot \log_2(M_{opt}[k]) &= \frac{R}{N}
\end{align*}
\]

where \( \lambda_{\min}(B) \) denotes the smallest singular value of matrix \( B, \) \( M_{opt}[k] \) is the symbol constellation used to modulate the \( p_{opt}[k] \) spatial-multiplexing data streams, such that the rate constraint of \( R/N \) per-subcarrier is fulfilled. The mode selection is carried out at the receiver, where the perfect and complete CSI is available. The resulting optimal number of spatial multiplexing streams, to be used on each subcarrier, \( \{p_{opt}[k]\}_{1 \leq k \leq N} \) are then fed-back to the transmitter together with the indexes of the \( U \) pilot precoders in \( F. \) Finally, the transmitter enforces the optimal spatial multiplexing mode, \( \{p_{opt}[k], M_{opt}[k]\} \), on each subcarrier \( k. \)

7. PERFORMANCE RESULTS

In this section, we assess the performance of our multi-mode precoding for spatial multiplexing MIMO-OFDM system with limited feedback, where the geodesic interpolation is used to reconstruct the unitary precoders on all subcarriers based on the feedback of pilot quantized precoders on a limited set of subcarriers. In order to do that, we consider a set-up consisting of a 2-antenna transmitter and a 2-antenna receiver at a rate of \( R = 64 \) Mbps. This rate corresponds to 2 QPSK-modulated streams per subcarrier over \( N = 64 \) subcarriers. Furthermore, we used the MIMO channel model provided by the IEEE 802.11 TGN [17] assuming the following assumptions: channel model B and F for the downlink and non-line-of-sight propagation, antenna spacings at the transmitter and the receiver are equal \( \lambda, \) where \( \lambda \) is the carrier wavelength at 5.2 GHz and a sampling rate of 20 MHz. At this sampling rate, channel model B (rms delay spread 15 ns) exhibits \( L = 10 \) samples, whereas channel model F (rms delay spread 150 ns) has a length \( L = 29. \) Finally, every point of the simulation results was obtained by averaging over more than 100 channel realizations.

The performance of the proposed interpolation obviously depends on the number of used pilot precoders \( U. \) Indeed, the reconstruction capability of the interpolation will depend on how the number of pilot precoders \( U \) compares to the length of the delay-spread channel \( L. \) For \( U \geq L, \) the interpolation will succeed in reconstructing the precoders based on the pilots. Whereas, when \( U < L, \) the interpolation will make errors on the precoders and will consequently lead to a degradation of the BER performance. Nevertheless, the latter case is the most relevant, as it enables the highest reduction in feedback requirements. Consequently, the illustrated performance results are dedicated to scenarios where \( U < L. \) Figure 1 depicts the average BER performance of our geodesic-interpolated multi-mode quantized precoding for IEEE 802.11 TGN channel ‘B’. It also compares this performance to those of the complete-CSI multi-mode unitary and min-MSE unquantized precoding. The latter solution corresponds to the optimal MMSE solution that also employs power loading across the spatial modes. Interestingly, our proposed solution exhibits only a 1 dB SNR loss at BER = \( 10^{-4} \) with respect to the perfect CSI solutions, while only requiring the feedback of the indexes on 8 pilot precoders out of a quantized precoder codebook \( F \) of cardinality \( card(F) = 4. \) More specifically, our solution entails \( 2 \times 8 \) bits to feedback the information related to the \( U = 8 \) pilots and 64 bits to feedback the \( p_{opt} \) values on the \( N = 64 \) subcarriers, leading to a total feedback of 80 bits. Clearly, the feedback overhead is dominated by the information about \( \{p_{opt}[k]\}_{k=1}^{N}. \) This feedback is justified by the significant performance improvement provided by multi-mode optimization over unoptimized solutions, as shown in Figure 1. The reduction of this feedback overhead is the topic of our current investigation. Figure 2 further confirms that similar performance results and feedback conclusions are obtained for channels with larger delay spread, such as IEEE 802.11 TGN channel ‘F’. Interestingly, our interpolation-based multi-mode solution is still shown to reasonably outperform its unoptimized counterparts with as few as \( U = 10 \) pilots, while the channel length is \( L = 29, \) and a codebook \( F \) of cardinality \( card(F) = 4. \) In this case, the total amount of feedback equals 84 bits.

8. CONCLUSIONS

We have proposed a novel multi-mode precoding architecture for MIMO-OFDM system with limited feedback. This architecture combines 2 key features to reduce the feedback requirements with minimum BER performance degradation. On the one hand, an efficient quantization of the unitary right singular matrices, based on a quantized codebook approach. On the other hand, a Stiefel-interpolator that is able to reconstruct the precoding matrices on all the subcarriers based on the feedback of the quantized precoders
Fig. 2. Average uncoded BER comparison for the considered (2,2) MIMO-OFDM set-up at rate 64 Mbps with channel F and 10 pilot precoders and \( \text{card}(F) = 4 \)

on a fraction of the subcarriers. The mode selection, which has been carried out at the receiver and fed-back to the transmitter, is then enforced on the reconstructed quantized right singular matrices on all subcarriers. Finally, we have shown that the proposed multi-mode precoding architecture exhibits good performance at reasonable feedback requirements.

9. REFERENCES


