Energy-Efficient Distributed Spectrum Sensing for Cognitive Sensor Networks

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Abstract—Reliability and energy consumption in detection are key objectives for distributed spectrum sensing in cognitive sensor networks. In conventional distributed sensing approaches, although the detection performance improves with the number of radios, so does the network energy consumption. We consider a combined sleeping and censoring scheme as an energy efficient spectrum sensing technique for cognitive sensor networks. Our objective is to minimize the energy consumed in distributed sensing subject to constraints on the detection performance, by optimally choosing the sleeping and censoring design parameters. The constraint on the detection performance is given by a minimum target probability of detection and a maximum permissible probability of false alarm. Depending on the availability of prior knowledge about the probability of primary user presence, two cases are considered. The case where a priori knowledge is not available defines the blind setup; otherwise the setup is called knowledge-aided. By considering a sensor network based on IEEE 802.15.4/ZigBee radios, we show that significant energy savings can be achieved by the proposed scheme.

Index Terms—distributed spectrum sensing, cognitive sensor networks, detection and fusion performance.

I. INTRODUCTION

Recent advances in wireless communication technologies and services have created a tremendous demand for radio spectrum. Radio spectrum has largely been managed under a licensed approach that has led to the current day scarcity in spectrum. However a number of recent spectrum measurements [1], [2], [3] has shown that licensed spectrum is under-utilized and that there exist spectrum portions unused over space and time. To promote utilization of such spectrum portions, dynamic spectrum sharing models based on cognitive radios have been proposed [4]. Spectrum regulations [5], [6] are underway to promote such technologies for secondary spectrum sharing of licensed spectrum. In its recent Report and Order [5], the FCC permitted the operation of networks consisting of low-power portable devices and sensors in the VHF-UHF band. The FCC is also in the process of seeking spectrum sharing of licensed spectrum. In its recent Report and Order [5], the FCC permitted the operation of networks consisting of low-power portable devices and sensors in the VHF-UHF band. The FCC is also in the process of seeking spectrum sharing of licensed spectrum. In its recent Report and Order [5], the FCC permitted the operation of networks consisting of low-power portable devices and sensors in the VHF-UHF band. The FCC is also in the process of seeking spectrum sharing of licensed spectrum. In its recent Report and Order [5], the FCC permitted the operation of networks consisting of low-power portable devices and sensors in the VHF-UHF band.

Cognitive radios achieve secondary spectrum access while limiting harmful interference to licensed primary users. To achieve this, spectrum sensing of radio channels is employed to identify channels that may be vacant. Transmission is then limited on channels determined to be empty in order to avoid interference with primary users. Reliable determination of empty channels is thus a critical problem.

The hidden terminal problem as well as fading effects can adversely affect the performance reliability of a cognitive radio. It has been shown that in a network of cognitive radios, distributed spectrum sensing improves the detection performance [7], [8]. Distributed spectrum sensing alleviates these problems by exploiting the spatial diversity from multiple signal observations at spatially distributed sensors. While distributed sensing does improve detection reliability, the network energy consumed scales with the number of radios in conventional schemes [7], [8].

In this paper, we are interested in the problem of distributed spectrum sensing in cognitive sensor networks. We shall refer to a cognitive sensor network as a wireless network of low-power radios that gain secondary spectrum access following the cognitive radio paradigm discussed earlier. We are interested in devising energy efficient strategies for distributed sensing.

We consider a distributed spectrum sensing system comprising of a fusion center (FC) and a number of cognitive radios that carry out sensing in dedicated, periodic sensing slots. Energy detection, which is a common approach to the detection of unknown signals [9], [10], is used for channel sensing. The sensing results of each cognitive radio are collected at the FC, which makes a global decision on the occupancy of the channel using a fusion rule. Schemes based on soft and hard fusion have been considered in the past [8] (the reader is referred to [11] for an extensive treatment of distributed detection). It has been shown in [8] that the performance of hard fusion schemes is comparable to that of soft fusion schemes in a number of practical settings. We shall hence limit our attention to hard decision based spectrum sensing, since the energy cost of sending one bit per decision is smaller than sending multiple bits per decision for a soft decision scheme.

We propose a combination of sleeping and censoring as an energy-saving mechanism for spectrum sensing. In this scheme, when in sleep mode, each radio switches off its sensing transceiver and incurs no observation costs or transmission costs. Censoring involves transmitting detection results only when they are in a certain information region. Our goal is to minimize the average energy incurred by the cognitive sensor network to perform spectrum sensing while maintaining a global detection performance by determining the optimum sleeping and censoring parameters. The constraints on the detection performance are specified by a minimum...
target probability of detection and a maximum permissible probability of false alarm. We consider two cases based on the availability of prior knowledge about the probability of primary user presence. For the case that the prior probabilities are not available, a blind setup is defined. When the prior probabilities are available, a knowledge-aided setup is described. Systematic algorithms for obtaining the optimum sleeping and censoring parameters are proposed for both setups. We then consider a network of IEEE 802.15.4/ZigBee radios to evaluate the efficiency of our proposed scheme. Resulting simulation results show that large energy savings can be obtained in comparison to traditional spectrum sensing schemes.

Censoring has been considered in the context of wireless sensor networks and cognitive radios [13], [15], [16], [17], [18] and shown to be effective in saving energy. The design of censoring regions under different optimization settings related to the detection performance has been considered in [15]-[18] for minimization of the miss detection probability with constraints on the false alarm rate and the network energy consumption. Further, [15], [16] and [18] consider minimization of the detection error probability subject to the network energy consumption. The combination of sleeping and censoring was considered in [14], with the goal of maximizing the mutual information between the state of signal occupancy and the decision state of the fusion center. Censoring for cognitive radios is considered in [13] where a censoring decision rule similar to our scheme is employed to reduce the number of bits sent to the fusion center and so the bandwidth occupancy of the cognitive radio network. Our scheme is different in three ways. First we consider a combination of sleeping and censoring and give closed-form analytical expressions for the probability of detection and false alarm. Second, we give a clear problem formulation and necessary algorithms to solve the problem in order to design the sensing parameters which is not given in [13]. Third, in [13], only the knowledge-aided setup is considered for analysis while we also consider the blind setup. Finally, the fusion center in [13] makes no decision in case it does not receive any results from the cognitive users which is ambiguous in the sense that the FC has to make a final decision about the presence (or absence) of the primary user. In this paper, if no results are reported to the FC, we assume that the primary user is not present. A sleeping technique is employed in [26] where the sleeping policy is controlled by learning from the past channel observations. As shall be shown, the optimization problems resulting from our work differ from these mentioned past works; we lay constraints on the detection performance while the energy consumption is minimized. Furthermore, a cluster-based and a confidence voting approach to energy efficient distributed sensing is proposed in [12]. In the cluster-based approach, a cognitive radio network is divided into several clusters based on their geometric location. Each cognitive radio sends its local decision to its assigned cluster head which makes a local cluster decision and sends it to the fusion center. This way the network energy consumption reduces due to the distance reduction by avoiding broadcasting every result to the fusion center directly. In the confidence voting approach, each user sends its local decision to the FC only if it is deemed confident enough. The secondary user looks for a consensus among the other users and if its result is in accordance with the majority opinion, it gains confidence else its confidence level decreases. Each user can send its result to the FC only if its confidence level is above a certain threshold. However, these approaches are mainly protocol based schemes and the detection technique as well as the underlying problem formulation for system design parameters are not given. Our proposed technique can be combined with the technique proposed in [12] to achieve even more energy savings.

The remainder of the paper is organized as follows. In Section II, we describe distributed spectrum sensing based on sleeping and censoring and formulate energy-efficient distributed sensing as an optimization problem for the blind and knowledge-aided setups. Expressions for the global probability of detection and false alarm are then derived in Section III. In Section IV, the problem is analyzed and systematic algorithms are proposed to solve the underlying optimization problems for both setups. We present numerical and simulation results to show the energy savings obtained by the proposed scheme in Section V. Conclusions are drawn in Section VI.

II. SYSTEM MODEL

The considered distributed spectrum sensing system comprises of $N$ cognitive sensors and an FC in a parallel distributed fusion configuration as shown in Fig. 1. In such a configuration, each of the radios makes its own local decision and sends the result to the FC. The FC combines these local decisions according to a certain rule and makes the final decision by solving a binary hypothesis testing problem, i.e. the FC determines whether a primary system is transmitting, given by hypothesis $\mathcal{H}_1$, or not, given by hypothesis $\mathcal{H}_0$. Each radio is controlled by two policies: (i) a sleeping policy determines whether or not it is awake, and (ii) a censoring policy determines whether or not it transmits its detection result, given that it is awake. Denote $\mu$ to be the sleeping rate, i.e. the probability that a radio is turned off. Each radio $i$ that is awake performs detection in a dedicated sensing slot using $T_0$ observation samples, denoted by $x_i[k]$, $k = 1, 2, ..., T_0$. Each observation sample $x_i[k]$ follows the data model,

$$x_i[k] = \begin{cases} s_i[k] + n_i[k] & \text{under } \mathcal{H}_0 \\ n_i[k] & \text{under } \mathcal{H}_1 \end{cases}$$

where the primary user’s signal and the noise at the $i$-th radio are denoted by $s_i[k]$ and $n_i[k]$ respectively. The noise is assumed to be an i.i.d. Gaussian random process with zero mean and variance $\sigma_n^2$ and the signal is assumed to be deterministic. An energy detector is employed by each cognitive sensor that calculates the accumulated energy over $T_0$ observation samples. The received energy collected over the $T_0$ observation samples at the $i$-th radio is given by

$$E_i = \sum_{k=1}^{T_0} \sigma_n^2[k].$$

Afterwards a censoring policy is employed at each radio [15], [18]. Censoring thresholds $\lambda_1$ and $\lambda_2$ are applied at each of the radios. The range $\lambda_1 < E_i < \lambda_2$ is called the censoring
region. At the $i$-th radio, the local censoring decision rule is given as
\[
\begin{cases}
\text{send 1, declaring } H_1 & \text{if } E_i \geq \lambda_2, \\
\text{no decision} & \text{if } \lambda_1 < E_i < \lambda_2, \\
\text{send 0, declaring } H_0 & \text{if } E_i \leq \lambda_1.
\end{cases}
\]
(3)

In practice the average received signal-to-noise ratio (SNR) at each cognitive radio is different. However, the system parameter design becomes very difficult and even analytically intractable for different SNRs. Particularly in our scheme, the problem becomes NP-complete. For analytical tractability, we assume that the received signal-to-noise ratio (SNR) at each radio is the same, denoted by $\gamma$. Such an assumption still allows us to gain valuable insight into the design of censoring and sleeping parameters. This has also been considered in [25] which presents an experimental study of cooperative spectrum sensing where the received SNR at each cognitive radio is assumed to be the same and it is shown that cooperative sensing still improves the detection performance of the cognitive network. Following this assumption, the probabilities of false alarm and detection for each radio are the same, denoted respectively by $P_f$ and $P_d$. It is well known [10] that under the model (1)-(2), $E_i$ follows a central chi-square distribution with $2T_0$ degrees of freedom under $H_0$ and a non-central chi-square distribution with $2T_0$ degrees of freedom and non-centrality parameter $2\gamma$ under $H_1$.

Based on the above decision rule, the local probabilities of false alarm and detection can be respectively written as
\[
P_f = P_r(E_i \geq \lambda_2 | H_0) = \frac{\Gamma(\frac{T_0}{2}, \frac{\lambda_2}{\gamma})}{\Gamma(\frac{T_0}{2})},
\]
(4)
and
\[
P_d = P_r(E_i \geq \lambda_2 | H_1) = Q_{\Gamma_0}(\sqrt{2\gamma}, \sqrt{\lambda_2}),
\]
(5)
where $\Gamma(a, x)$ is the incomplete gamma function given by $\Gamma(a, x) = \int_x^\infty t^{a-1}e^{-t}dt$, with $\Gamma(a, 0) = \Gamma(a)$ and $Q_{\Gamma_0}(a, x) = \frac{1}{\pi} \int_x^\infty t^{a-1}e^{-\frac{t^2}{2}}I_{a-1}(t)dt$, with $I_{a-1}(\cdot)$ being the modified Bessel function of the first kind and order $u - 1$.

Denote $C_a$ and $C_t$ to be the energy consumed by the $i$-th radio in sensing and transmission, respectively. Our cost function is then given by the average energy consumed for distributed sensing in the network,
\[
C_T = (1 - \mu) \sum_{i=1}^{N} (C_a + C_t(1 - \rho)),
\]
(6)
where $\rho = Pr(\lambda_1 < E_i < \lambda_2)$ is denoted to be the censoring rate.

We shall assume that $\mu \neq 0$ and $\rho \neq 0$. The sensing energy $C_a$ constitutes the energy consumed in listening and collecting the $T_0$ observation samples, as well as the energy required for making a local decision. The transmission energy $C_t$ is the energy required to transmit the one-bit local decision to the FC.

Denote $Q_D$ and $Q_p$ to be the respective global probability of detection and false alarm. The target detection performance is then quantified by: $Q_p \leq \alpha$ and $Q_D \geq \beta$. Here, $\alpha$ and $\beta$ are pre-specified detection design parameters. In practice, it is desirable to have $\alpha$ close to zero and $\beta$ close to unity. These conditions respectively ensure that the cognitive sensor network can, exploit empty channels and that primary users are not interfered with. Our goal is to determine the optimum sleeping rate $\mu$ and the censoring thresholds $\lambda_1$ and $\lambda_2$ such that $C_T$ in (6) is minimized subject to the constraints $Q_D \leq \alpha$ and $Q_P \geq \beta$. Note from (8) that $\rho$ can be written as a function of $\lambda_1$ and $\lambda_2$. Hence our optimization problem can be formulated as follows:
\[
\min_{\mu, \lambda_1, \lambda_2} C_T \\
\text{s.t. } Q_P \leq \alpha, \ Q_D \geq \beta.
\]
(7)

Depending on the prior knowledge about the respective prior probabilities, $\pi_0 = Pr(H_0)$ and $\pi_1 = Pr(H_1)$, of the hypotheses $H_0$ and $H_1$, we consider two different cases.

A. Blind Problem Formulation

First, we assume that $\pi_0$ and $\pi_1$ are unknown, and that $\pi_1$ is much smaller than $\pi_0$, reflecting channel under-utilization. In this case, we can follow the definition of [18] for the censoring rate under the blind Neyman-Pearson (NP) setup
\[
\rho^{NP} = Pr(\lambda_1 < E_i < \lambda_2 | H_0).
\]

Using (4), we may write $\rho^{NP}$ as
\[
\rho^{NP} = \frac{\Gamma(T_0, \frac{\lambda_2}{\gamma})}{\Gamma(T_0)} - \frac{\Gamma(T_0, \frac{\lambda_1}{\gamma})}{\Gamma(T_0)}.
\]
(8)

Denoting $Q_D^{NP}$ and $Q_P^{NP}$ to be the respective global probability of detection and false alarm under the blind setup, (7) becomes
\[
\min_{\mu, \lambda_1, \lambda_2} C_T^{NP} \\
\text{s.t. } Q_P^{NP} \leq \alpha, \ Q_D^{NP} \geq \beta.
\]
(9)

B. Knowledge-Aided Problem Formulation

Here, we assume that $\pi_0$ and $\pi_1$ are known. In practice, estimates of $\pi_0$ and $\pi_1$ can be obtained via spectrum measurements. In this case, we can follow the definition of [18] for the censoring rate under the knowledge-aided Bayesian (B) setup
\[
\rho^B = Pr(\lambda_1 < E_i < \lambda_2) = \pi_0 Pr(\lambda_1 < E_i < \lambda_2 | H_0) + \pi_1 Pr(\lambda_1 < E_i < \lambda_2 | H_1) = \pi_0 \delta_0 + \pi_1 \delta_1
\]
(10)
where $\delta_0$ and $\delta_1$ can be written using (4) and (5) as

$$
\delta_0 = Pr(\lambda_1 < E_i < \lambda_2 | \mathcal{H}_0)
= \frac{\Gamma(T_0, \frac{\lambda_2}{2}) - \Gamma(T_0, \frac{\lambda_1}{2})}{\Gamma(T_0)}.
$$

$$
\delta_1 = Pr(\lambda_1 < E_i < \lambda_2 | \mathcal{H}_1)
= Q_{T_0}(\sqrt{2T_1}, \sqrt{\lambda_1}) - Q_{T_0}(\sqrt{2T_1}, \sqrt{\lambda_2}).
$$

Denote $Q_{\mathcal{D}}^B$ and $Q_{\mathcal{F}}^B$ to be the respective global probability of detection and false alarm under the knowledge-aided setup. Hence, our optimization problem becomes

$$
\min_{\mu, \lambda_1, \lambda_2} C^B_T
\text{s.t. } Q_{\mathcal{D}}^B \leq \alpha, \ Q_{\mathcal{F}}^B \geq \beta.
$$

In the following section, we derive analytically the expressions for $Q_{\mathcal{D}}^N$, $Q_{\mathcal{F}}^N$, $Q_{\mathcal{D}}^B$ and $Q_{\mathcal{F}}^B$.

### III. DETECTION PERFORMANCE ANALYSIS

Each cognitive radio that is awake listens to the channel in dedicated sensing slots. An awake cognitive radio computes the received signal energy and locally decides on the presence or absence of the licensed system based on the decision rule in (3). If it comes up with a decision, then it sends its decision result to the FC. The FC employs an OR rule to make the final decision denoted by $D_{FC}$. That is, $D_{FC} = 1$ if the FC receives at least one local decision declaring 1, else $D_{FC} = 0$. Let the number of awake cognitive radios be $K$, and let $L$ out of $K$ such cognitive radios send their decision to the FC.

The probability of false alarm for the blind setup, $Q_{FP}^B$, can now be written as

$$
Q_{FP}^B = Pr(D_{FC} = 1, L \geq 1, K \geq 1 | \mathcal{H}_0)
= \sum_{K=1}^{N} Pr(D_{FC} = 1, L \geq 1, K \geq 1 | \mathcal{H}_0)
= \sum_{K=1}^{N} Pr(K | \mathcal{H}_0) Pr(D_{FC} = 1, L \geq 1 | \mathcal{H}_0, K)
= \sum_{K=1}^{N} \binom{N}{K} \mu^{N-K} (1-\mu)^K
\times \sum_{L=1}^{K} Pr(D_{FC} = 1, L | \mathcal{H}_0, K)
= \sum_{K=1}^{N} \binom{N}{K} \mu^{N-K} (1-\mu)^K
\times \sum_{L=1}^{K} Pr(L | \mathcal{H}_0, K) Pr(D_{FC} = 1 | \mathcal{H}_0, K, L)
= \sum_{K=1}^{N} \binom{N}{K} \mu^{N-K} (1-\mu)^K
\times \sum_{L=1}^{K} \binom{K}{L} \delta^{K-L} (1-\delta)^L [1 - (1-P_d)^L]
= 1 - \{1 - (1-\mu)(1-\delta)P_d\}^N,
$$

where $\delta = P(\lambda_1 < E_i < \lambda_2 | \mathcal{H}_1)$ and $P_d$ is given by (5). This also can be explained by the OR rule based global probability of detection when considering $P_{\mathcal{D}}^N = (1-\mu)(1-\delta)P_d$ to be the local probability of detection including the censoring and sleeping policies.

Denoting $P_{\mathcal{F}}^B = (1-\mu)(1-\delta_0)P_f$ to be the local probability of false alarm including the censoring and sleeping policies, the global probability of false alarm for the knowledge-aided scenario, $Q_{FP}^B$, can be written as

$$
Q_{FP}^B = Pr(D_{FC} = 1, L \geq 1, K \geq 1 | \mathcal{H}_0)
= \sum_{K=1}^{N} Pr(D_{FC} = 1, L \geq 1, K \geq 1 | \mathcal{H}_0)
= \sum_{K=1}^{N} Pr(K | \mathcal{H}_1) Pr(D_{FC} = 1, L \geq 1 | \mathcal{H}_1, K)
= \sum_{K=1}^{N} \binom{N}{K} \mu^{N-K} (1-\mu)^K
\times \sum_{L=1}^{K} Pr(D_{FC} = 1, L | \mathcal{H}_1, K)
= \sum_{K=1}^{N} \binom{N}{K} \mu^{N-K} (1-\mu)^K
\times \sum_{L=1}^{K} \binom{K}{L} \delta^{K-L} (1-\delta)^L [1 - (1-P_f)^L]
= 1 - \{1 - (1-\mu)(1-\delta)P_d\}^N.
$$
Denoting $P_{dl}^B = (1 - \mu)(1 - \delta_1)P_d$ to be the local probability of detection including the censoring and sleeping policies, the global probability of detection for the knowledge-aided scenario, $Q_{dl}^B$, can be derived in a similar way. We obtain

$$Q_{dl}^B = \Pr(D_{FC} = 1, L \geq 1, K \geq 1\mid \mathcal{H}_1) = 1 - \left(1 - P_{dl}^B\right)^N = 1 - \left(1 - (1 - \mu)(1 - \delta_1)P_d\right)^N, \quad (18)$$

where $P_d$ is given by (5).

In the following section, we analyze the optimization problems (9) and (13) given the expressions for the constraints derived in this section and we propose an algorithm to solve them.

**IV. PROBLEM ANALYSIS**

In this section, (9) and (13) are analyzed in order to find a systematic solution for the system parameters, namely the sleeping rate and censoring thresholds for the two setups.

Before going forward with the problem analysis we introduce the following lemma, which is used to simplify the optimization problems in the subsequent subsections.

**Lemma 1**: If the feasible set of (7) is not empty, then $\lambda_1 = 0$ in the feasible set of the problem.

**Proof**: Denote $\mathcal{M}$ to be the feasible set of (7). Assume $\exists (\mu^*, \lambda_1^*, \lambda_2^*) \in \mathcal{M}$ where $\lambda_1^* \neq 0$ is the lowest $\lambda_1 \in \mathcal{M}$. Inserting $\mu^*$ in $Q_B$ and $Q_F$ we define the following problem with $\mathcal{M}$ denoting its corresponding feasible set,

$$\max_{\lambda_1, \lambda_2} Q_B \quad \text{s.t.} \quad Q_F \leq \alpha.$$  

(19)

Denoting the respective local probability of false alarm and detection including censoring and sleeping policies by $P_{fl}$ and $P_{dl}$. We obtain

$$\max_{\lambda_1, \lambda_2} 1 - \left(1 - P_{fl}\right)^N \quad \text{s.t.} \quad 1 - \left(1 - P_{fl}\right)^N \leq \alpha.$$  

(20)

and after simplifications (20) becomes

$$\max_{\lambda_1, \lambda_2} P_{dl} \quad \text{s.t.} \quad P_{fl} \leq 1 - (1 - \alpha)^{1/N}.$$  

(21)

Since $\delta_0 = \rho NP$ and $\delta_1 = \delta$, without loss of generality, we can denote $P_{fl} = (1 - \mu)(1 - \rho NP)P_f$ and $P_{dl} = (1 - \mu)(1 - \delta)P_d$. Since $\frac{\partial P_{fl}}{\partial \mu} = -(1 - \mu)P_f (1 - \rho NP) \geq 0$ (where we used the fact that $\frac{d\rho}{d\mu} \leq 0$), if $(\lambda_1, \lambda_2) \in \mathcal{M}$, then $(\lambda_1 = 0, \lambda_2) \in \mathcal{M}$. Therefore, $\exists (\lambda_1 = 0, \lambda_2) \in \mathcal{M}$. Further, it is clear that $(\mu^*, 0, \lambda_2^*) \in \mathcal{M}$ which is a contradiction with our assumption that $\lambda_1^* \neq 0$. Hence, if $\mathcal{M} \neq 0$ then $\lambda_1 = 0 \in \mathcal{M}$. □

**A. Blind Setup**

Based on (15) and (16), (9) can be written as

$$\min_{\mu, \lambda_1, \lambda_2} \left(1 - \mu\right) \sum_{i=1}^N [C_{si} + C_{ei} (1 - \rho^\text{NP})] \quad \text{s.t.} \quad 1 - \left[1 - (1 - \mu)(1 - \rho NP)P_f^*\right]^N \leq \alpha$$

$$1 - \left[1 - (1 - \mu)(1 - \delta_1)P_d\right]^N \geq \beta.$$  

(22)

Since $C^\text{NP}^T(\rho^\text{NP}) = \frac{\partial C^\text{NP}^T}{\partial \rho^\text{NP}} \leq 0$ and $\rho^\text{NP}^T(\lambda_1) = \frac{\partial C^\text{NP}^T}{\partial \lambda_1} \leq 0$, we obtain $C^\text{NP}^T(\lambda_1) = C^\text{NP}^T(\rho^\text{NP})^T(\rho^\text{NP}^T(\lambda_1)) \geq 0$. Therefore, the optimal $C^\text{NP}^T$ is attained for the lowest $\lambda_1$ in the feasible set of the problem that based on Lemma 1 is equal to 0. Using this result, we can relax one of the arguments of the problem. Furthermore, when $\lambda_1 = 0$, we obtain $1 - \rho^T = P_f$ and $1 - \delta = P_d$. Thus, after some simplifications and using the result, we can relax one of the arguments of the problem. Hence, if $\lambda_2 = 2 \Gamma^{-1}[T_0, \Gamma(T_0)P_f])$, the problem (22) can be written as

$$\min_{\mu, P_f} \left(1 - \mu\right) \sum_{i=1}^N [C_{si} + C_{ei} P_f] \quad \text{s.t.} \quad \frac{1}{(1 - \mu)^2} - \frac{1}{(1 - \mu)P_f^2} - \frac{1}{1 - (1 - \alpha)^{1/N}} \leq 0.$$  

(23)

In the above problem, the objective function and the function $\left(1 - \mu\right)P_f^2$ are convex with respect to $\mu$ and $P_f$ individually, but not jointly. We now prove that $\frac{1}{(1 - \mu)^2}$ is also convex in $\mu$ and $P_f$ individually. The second derivative of $\frac{1}{(1 - \mu)^2}$ is $\frac{2}{(1 - \mu)^3}$, which is convex with respect to $\mu$. It is well known that for a LR test continuous test, $P_d$ is concave in $P_f$ [11, p 14] and so is log-concave in $P_f$ (note that the energy detector becomes a LRT detector for the Gaussian signals). Since the product of two log-concave functions is log-concave, $P_d$ is log-concave, thus, $\frac{1}{P_d}$ is convex with respect to $P_f$.

Although (23) is not a standard convex optimization problem, we can still exploit the individual convexity of the problem in $\mu$ and $P_f$ for a systematic solution. Therefore, for solving the problem, we solve the resulting convex problem to find $P_f$ (or $\mu$) for a given $\mu$ (or $P_f$) over the range of $0 < \mu < 1$ ($0 < P_f < 1$). Finally, we need to locate the minimum $C^\text{NP}^T$ and its corresponding parameters, $P_f$ and $\mu$ using an exhaustive search. Further, we can also employ standard systematic optimization tools such as alternating optimization, leading to a local instead of a global solution.

**B. Knowledge-Aided Setup**

To analyze (13), it is more convenient to rewrite it in the following format

$$\min_{\mu, \lambda_1, \lambda_2} \left(1 - \mu\right) \sum_{i=1}^N [C_{si} + C_{ei} (1 - \rho^\text{NP})] \quad \text{s.t.} \quad 1 - \left[1 - (1 - \mu)(1 - \delta_0)P_f\right]^N \leq \alpha$$

$$1 - \left[1 - (1 - \mu)(1 - \delta_1)P_d\right]^N \geq \beta.$$  

(24)

Similar to the blind setup we can prove that if the feasible set of (24) is not empty, then the optimal $C^B_{dl}$ is attained for $\lambda_1 = 0$. Using this result, we can relax one of the arguments of the problem. Thus, the new problem becomes

$$\min_{\mu, \lambda_2} \left(1 - \mu\right) \sum_{i=1}^N [C_{si} + C_{ei} (1 - \rho^\text{NP})] \quad \text{s.t.} \quad 1 - \left[1 - (1 - \mu)(1 - \delta_0)P_f\right]^N \leq \alpha$$

$$1 - \left[1 - (1 - \mu)(1 - \delta_1)P_d\right]^N \geq \beta.$$  

(25)
When $\lambda_1 = 0$, we obtain

$$
\begin{align*}
1 - \delta_1 &= P_f, \\
1 - \delta_1 &= P_d.
\end{align*}
$$

(26)

Hence, (24) is given by

$$
\begin{align*}
\min_{\mu, C_2} (1 - \mu) \sum_{i=1}^{N} [C_{s_i} + C_{t_i}(\pi_0 P_f + \pi_1 P_d)] \\
\text{s.t. } &1 - (1 - (1 - \mu)P_d^2]_{1}^{N} \leq \alpha \\
&1 - (1 - (1 - \mu)P_d^2]_{1}^{N} \geq \beta.
\end{align*}
$$

(27)

After rewriting (27) in the standard optimization problem format [19], we obtain

$$
\begin{align*}
\min_{\mu, P_f} (1 - \mu) \sum_{i=1}^{N} [C_{s_i} + C_{t_i}(\pi_0 P_f + \pi_1 P_d)] \\
\text{s.t. } &1 - (1 - \mu)P_f^2 \leq 1 - (1 - \alpha)^{1/N} \\
&\frac{1}{(1 - \mu)P_f^2} \leq \frac{1}{1 - (1 - \beta)^{1/N}}.
\end{align*}
$$

(28)

Similar to the blind setup, we can show that the constraints are convex with respect to $\mu$ and $P_f$ individually, but the objective function is not convex in $P_f$. However, as we will show in the following, the problem can still be solved systematically.

Assume that $\mu$ is fixed to $\mu^*$. Then (28) will reduce to the following problem

$$
\begin{align*}
\min_{P_f} (1 - \mu^*) \sum_{i=1}^{N} [C_{s_i} + C_{t_i}(\pi_0 P_f + \pi_1 P_d)] \\
\text{s.t. } &P_f^2 \leq 1 - (1 - \alpha)^{1/N} \\
&\frac{1}{P_f^2} \leq \frac{1}{1 - (1 - \beta)^{1/N}}.
\end{align*}
$$

(29)

Defining $F(\lambda_2) = \frac{r(T_0, \lambda_2)}{a(T_0)}$, we can write $P_f$ as $P_f = Q_{T_0}(\sqrt{2\gamma}, \sqrt{2F^{-1}(P_f)})$. Calculating the derivative of $C_{f_0}^P$ with respect to $P_f$, we find that

$$
\frac{\partial C_{f_0}^P}{\partial P_f} = (1 - \mu^*) \sum_{i=1}^{N} C_{t_i}(\pi_0 P_f + \pi_1 P_d) - \left(1 - \mu^*\right) \sum_{i=1}^{N} C_{t_i}(\pi_0 P_f + \pi_1 P_d) \\
+ \pi_0 \sum_{i=1}^{N} C_{t_i}(\pi_0 P_f + \pi_1 P_d) \\
+ \frac{\partial \pi_0 F}{\partial P_f} \geq 0.
$$

Therefore we can write (29) as follows

$$
\begin{align*}
\min_{P_f} &\sum_{i=1}^{N} [C_{s_i} + C_{t_i}(\pi_0 F(G^{-1}(\xi)) + \pi_1 \xi)] \\
\text{s.t. } &P_f^2 \leq \frac{1}{1 - (1 - \alpha)^{1/N}} \\
&\frac{1}{P_f^2} \leq \frac{1}{1 - (1 - \beta)^{1/N}}.
\end{align*}
$$

(30)

Here, we have to note that $\mu^*$ cannot be chosen arbitrarily. Assuming $\alpha' = 1 - (1 - \alpha)^{1/N}$ and $\beta' = 1 - (1 - \beta)^{1/N}$, the detection probability constraint is generally larger than the false alarm rate constraint $\beta' > \alpha'$. So regarding that $P_f^2 \leq 1$, we thus have $\frac{\beta'}{\alpha'} \leq 1$. Therefore, we obtain $\mu^* \leq 1 - \beta'$ and thus, $\mu_{\max} = 1 - \beta'$.

Looking at (30) we can find that

$$
F(G^{-1}(\sqrt{\beta' - 1/\mu^*})) \leq P_f \leq \sqrt{\alpha' - 1/\mu^*}
$$

(31)

where $G(\lambda_2) = Q_{T_0}(\sqrt{2\gamma}, \sqrt{2\gamma})$. Thus, we find that for every $0 < \mu^* \leq \mu_{\max}$, $P_f = F(G^{-1}(\sqrt{\beta' - 1/\mu^*}))$. Therefore, our minimization problem for $0 < \mu \leq \mu_{\max}$ reduces to the following unconstrained line search problem

$$
\min_{\mu} (1 - \mu) \sum_{i=1}^{N} [C_{s_i} + C_{t_i}(\pi_0 F(G^{-1}(\xi)) + \pi_1 \xi)]
$$

(32)

where $\xi = \sqrt{\beta' - 1/\mu}$. Looking carefully at (32), we find that we can use the same optimization problem for the blind setup by considering $\pi_0 = 1 (\pi_1 = 0)$. In other words, the blind setup is just a special case of the knowledge-aided setup. This is the approach that we will adopt in the simulations for both setups.

V. NUMERICAL AND SIMULATION RESULTS

A. Numerical Analysis

We first numerically analyze the problem for different scenarios. A network of 5 cognitive radios with the same sensing and transmission energy is employed. In this network, each cognitive radio experiences an SNR of 10 dB. The aim is to analyze how the optimal parameters change with respect to different detection performance constraints. In one scenario, the sensing and transmission energies are assumed to be the same and in the other one the transmission energy is assumed to be 10 times larger than the sensing energy. We note that for the case where the sensing energy is 10 times larger than the transmission energy, we obtain results very close to the case where the sensing energy is equal to the transmission energy and hence these results are not shown.

In Fig. 2, the optimal censoring and sleeping rates are shown for $\alpha = 0.1$ and $0.8 \leq \beta \leq 0.99$. It is shown that as the transmission energy increases with respect to the sensing energy, the censoring rate increases while the sleeping rate decreases. The reason is that as the transmission energy becomes significantly larger compared to the sensing energy, the total transmission energy has to be reduced more than the sensing energy.

Fig. 3 shows the optimal censoring and sleeping rates for $0.01 \leq \alpha \leq 0.1$ and $\beta = 0.9$. Similar to the previous case, it is shown that the optimal censoring rate increases as the transmission energy increases with respect to the sensing energy while the sleeping rate decreases.

B. Case Study for IEEE 802.15.4/ZigBee

Here, a case study is considered in order to verify the performance of the proposed combined sleeping and censoring scheme. A Chipcon CC2420 transceiver based on the IEEE 802.15.4/ZigBee standard [20] is considered to compute the energy consumption in sensing and transmission. This low-power radio with a data rate up to 250 Kbps is aimed to work as a wireless personal area network up to ranges of 100 m. Our cognitive sensor network comprises of such radios arranged in a circular field with a radius of 70 m, uniformly distributed along the circumference with the FC located in the center. We model the wireless channel between the cognitive sensor and the FC using a free-space path loss model. This means that
the signal power attenuation is inversely proportional to the square of the distance $d$ between the transmitter and receiver.

The energy consumption analysis that is presented here is based on the transceiver model developed in [21]. The sensing energy for each decision consists of two parts: the energy consumption involved in listening over the channel and making the decision and the energy consumption of the signal processing part for modulation, signal shaping, etc. The former contribution depends on the number of samples taken during the detection time. We choose $T_0 = 5$, corresponding to a detection time of 1 µs. Considering the fact that the typical circuit power consumption of ZigBee is approximately 40 mW, the energy consumed for listening is approximately 40 nJ. The processing energy related to the signal processing part in the transmit mode for a data rate of 250 kbps, a voltage of 2.1 V, and current of 17.4 mA is approximately 150 nJ/bit. Since we use one bit per decision, the sensing energy of each cognitive sensor is $C_s = 190$ nJ [22], [23].

The transmitter dissipates energy to run the radio electronics and the power amplifier. Following the model in [21] and [24], to transmit one bit over a distance $d$, the radio spends:

$$C_t(d) = C_{t-elec} + e_{amp}d^2$$ (33)

where $C_{t-elec}$ is the transmitter electronics energy and $e_{amp}$ is the amplification required to satisfy a given receiver sensitivity level. Assuming a data rate of 250 kbps and a transmit power of 20 mW, $C_{t-elec} = 80$ nJ. The $e_{amp}$ to satisfy a receiver sensitivity of -90 dBm at an SNR of 10 dB is 40.4 pJ/m² [22], [23].

Every simulation result in this section is averaged over 1000 realizations. Two sets of values were chosen for the $\pi_0$ and $\pi_1$ a priori probabilities: $\pi_0 = 0.2, \pi_1 = 0.8$ and $\pi_0 = 0.8, \pi_1 = 0.2$. In Fig. 4, we show the energy consumed in spectrum sensing for different values of the probability of detection constraint, $\beta$. Here, $N = 5$, SNR = 10 dB and $\alpha = 0.1$. As is clear, a combined sleeping and censoring scheme consumes less than half the energy as would be consumed if a distributed spectrum sensing such as in [8] were employed. Furthermore, we see that when $\pi_0$ is much higher than $\pi_1$, the blind setup gives almost the same performance as the knowledge-aided setup.

In Fig. 5, we show the average energy consumed as the number of cognitive sensors in the network is increased. Here, $\alpha = 0.1$ and $\beta = 0.9$. Without sleeping or censoring, the energy consumed in spectrum sensing scales linearly with the number of cognitive sensors. However with a sleeping and censoring scheme, the energy consumption saturates to a level that is several orders of magnitude lower. We clearly...
Fig. 5. Energy scaling with number of cognitive sensors for different setups.

see that to attain the desired detection performance level, only a small fraction of the cognitive sensors need to participate in spectrum sensing. Again, it is shown that the blind setup gives a lower bound of the system energy consumption for a certain detection performance.

Fig. 6 shows the optimal censoring and sleeping rate for different values of the probability of detection constraint $\beta$ and $\alpha = 0.1$. Since the sensing energy of a ZigBee network is much higher than its transmission energy, the optimal value for the sleeping rate is attained at $\mu_{\max}$ for different values of $\beta$. That is why in Fig. 6, the sleeping rate is shown to have the same value for different a priori probabilities $\pi_0$ and $\pi_1$ as well as for the blind setup. However, it is shown that the censoring rate changes with the a priori probabilities. It is clear that the optimal censoring rate increases with $\pi_0$ and is the largest for the blind setup ($\pi_0 = 1$).

In Fig. 7, we finally show how the optimal censoring and sleeping rates change with respect to the number of users for $\alpha = 0.1$ and $\beta = 0.9$. For this figure, the blind setup is used for the simulations. It is shown that as the number of users increases, the optimal sleeping rate increases dramatically in order to keep the system energy consumption as stable as possible. However, the optimal censoring rate saturates after a limited number of users.

VI. SUMMARY AND CONCLUSIONS

We presented an energy efficient distributed spectrum sensing technique based on the combination of censoring and sleeping policies. Depending on the knowledge of the a priori probability of primary user presence, a Neyman-Pearson (blind setup) and Bayesian (knowledge-aided setup) formulation was obtained with the goal of minimizing the network energy consumption subject to a global detection performance constraint. We then derived analytical expressions for the global probabilities of detection and false alarm for each setup. In seeking a systematic solution for the obtained optimization problems, we showed that the resulting optimization problem can be reduced to an unconstrained line search problem for both setups.

Numerical results were presented with different scenarios regarding the sensing and transmission energies. It was shown that in case the transmission energy is much higher than the sensing energy, the optimal sleeping rate is higher than when the sensing and transmission energy are equal to each other. We then considered a case study with IEEE 802.15.4/ZigBee radios. It was shown that the network energy consumption is reduced significantly and almost becomes independent of the
number of cooperating cognitive radios, for a large number of radios. Note that we did not address the design of protocols employed in the cognitive sensor network - in particular, the medium access protocol that individual sensors use to transmit their detection results to the FC. Optimizing the design of the protocol jointly with the sensing and censoring policies could lead to additional energy savings. Further, our analysis was based on the OR hard fusion rule. The design of sleeping and censoring schemes with extensions to other fusion rules and soft fusion is a subject of further study.

REFERENCES